A *Rosen* Etymology

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'*When a thing has been said once, it is hard to say it differently.*' (*Aristotle*)
I composed this essay as a memorial to my mentor *Robert Rosen*, who had said it first.
Sicut patet per Biologum mathematicum...

This essay contains a few of my interpretations of *Robert Rosen*'s conception of Nature. I shall study the four notions that form the core of his whole-lifetime's scientific work: *simple system*, *mechanism*, *complex system*, and *organism*. Their set-theoretic interconnections culminate in *Rosen*'s new taxonomy of natural systems.

1. Introduction. – Let us start with a short episode from Chapt. VI of *Lewis Carroll*'s book '*Through the Looking-Glass, and What Alice Found There*' (1871):

'I don't know what you mean by «glory»', Alice said.
Humpty Dumpty smiled contemptuously. 'Of course you don't – till I tell you. I meant «there's a nice knock-down argument for you»!'
'But «glory» doesn't mean «a nice knock-down argument»', Alice objected.
'When I use a word', Humpty Dumpty said, in a rather scornful tone, 'it means just what I choose it to mean – neither more nor less.'
'The question is', said Alice, 'whether you can make words mean so many different things.'
'The question is', said Humpty Dumpty, 'which is to be master – that's all.'

Humpty's point of view is known in philosophy as *nominalism:* the doctrine that universals or abstract concepts are mere names without any corresponding 'reality'. The issue arises because, in order to perceive a particular object as belonging to a certain class, say 'organism', one must have a prior notion of the term 'organism'. Does this term, described by this prior notion, then have an existence independent of particular organisms? When a word receives a specific technical definition, does it have to reflect its prior notion, the common-usage sense of the word? Nominalism says no.

A closely related issue is a fallacy of misconstrual in logic known as *semantic equivocation*. This fallacy is quite common, because words often have several different meanings. A word may represent any one of several concepts, and the semantics of its usage are context-dependent. Errors arise when the different concepts with different consequences are mixed together as one. For a word that has a technical definition in addition to its everyday meaning, *non sequitur* may result when the distinction is blurred.

Confusion often ensues from a failure to clearly understand that words mean '*neither more nor less*' than what they are defined to mean, not what they are perceived to mean. This happens even in mathematics, where terms are usually more precisely defined than in other subjects. The most notorious example is the term 'normal', which appears in numerous mathematical subject areas to define objects with specific properties. In almost all cases (*e.g.*, normal vector, normal subgroup, normal operator), the normal subclass is nongeneric within the general class of objects; *i.e.*, what is defined as 'normal' is anything but normal in the common-usage sense of 'standard', 'regular', or 'typical'.

It is not my purpose here to discuss nominalism and semantic equivocation themselves. They are simply the philosophical and logical undertone of what follows. The thrust of this essay is, rather, the exploration of four concepts that form the core of *Robert Rosen*'s whole-lifetime's scientific work:

Simple System	Mechanism
Complex System	Organism

I shall study these (and other related) terms as *Rosen* defined them, explore the interconnections among these concepts, and examine whether they correspond to 'reality' (*i.e.*, the meanings of these terms in common usage). References are drawn from the *Rosen* tierce '*Organisms as Causal Systems which are not Mechanisms: an Essay into the Nature of Complexity*' [1], the posthumously published collection '*Essays on Life Itself*' [2], and, of course, the *Rosen* Trilogy '*Fundamentals of Measurement and Representation of Natural Systems*' [3], '*Anticipatory Systems*' [4], and '*Life Itself*' [5].

A juxtaposition of the four concepts appears in the introduction of [1]:

'*1. Our current system theory, including all that presently constitutes physics or physical science, deals exclusively with a very special class of systems that I shall call* simple systems *or* mechanisms.
2. Organisms, and many other kinds of material systems, are not mechanisms in this sense. Rather, they belong to a different (and much larger) class of systems, which we shall call complex.
3. Thus the relation between contemporary physics and biology is not, as everyone routinely supposes, that of general to particular.
4. To describe complex systems in general, and organisms a fortiori*, an entirely novel kind of mathematical language is necessary.*
5. *A simple system can only* approximate *to a complex one, locally and temporarily, just as,* e.g., *a tangent plane can only approximate to a nonplanar surface locally and temporarily. Thus in a certain sense, a complex system can be regarded as a kind of global limit of its approximating simple subsystems.*
6. *Complex systems, unlike simple ones, admit a category of final causation, or anticipation, in a perfectly rigorous and nonmystical way.*'

Before I begin studying these terms in detail, let me give a hint of what is to come. *Rosen*, in Chapter 20 of [2], stated thus: '*My usage of the term* complex *differs essentially from the way other authors define it*.'

2. The Dichotomy. – Just as life itself, the *Rosen* terminology evolves. I shall take the amendments and supersedences in the chronological order of the five major references: [3] < [4] < [1] < [5] < [2].

'System' is a basic undefined term, a primitive. It takes on the intuitive meaning of 'a collection of material or immaterial things that comprises one's object of study'.

'Natural system' is also a primitive. It, therefore, also assumes its intuitive meaning, 'a system of material things in the real world'. It is often synonymously referred to as 'physical system' and 'material system'. In Sect. 2.1 of [4], a natural system is characterized as follows:

- a natural system is a member or element of the external world,
- a natural system is a set of qualities, to which definite relations can be imputed.

A characterization is not a formal definition: simply a refinement of the usual intuitive meaning.

In Sect. 5.4 of [3], one finds this:

'*… a* [natural] *system is* simple *to the extent that a single description suffices to account for our interaction of the system; it is* complex *to the extent that this fails to be true.*'

Note that these are 'the beginning', preliminary definitions of the terms 'simple system' and 'complex system'. 'Simple' and 'complex' are used as attributes of natural systems, but the adjectives themselves are not defined (and therefore must have their common-usage meanings whenever they are *not* modifying the noun 'system'). From the outset, a dichotomy is established. A complex system is defined as a system that is not simple. The two categories of simple and complex systems are mutually exclusive. They are, indeed, complements of each other. I shall discuss the concept of 'complement' in more detail below (see *Chapt. 6*).

Note also that the term 'complexity' is never defined either. It is just used as the noun corresponding to the adjective 'complex', taking the common-usage meaning of 'the state or quality of being complex'.

Because of the *Rosen*-defined dichotomy, there are no 'degrees of complexity' when it comes to natural systems. According to the *Rosen* definition, a natural system is either simple or complex. One may, at best, make the trivial declarations that 'a simple system is simpler than a complex system' and that 'a complex system is more complex than a simple system'. But one cannot say, for example, that 'a complex natural system is *more complex* than another', when 'complex' is used as the *Rosen* attribute of natural systems.

The comparatives 'simpler' and 'more complex' have no *Rosen* definitions, and, therefore, must retreat to their common-usage meanings, *i.e.*, that of 'less complicated' and 'more complicated'. So, when one says one system is more complex than another, the complexity therein is treated as a measurable, observable property, *i.e.*, in the *von Neumann* sense (see, *e.g.*, Chapt. 2 of [2]). For example, one may say that an object B is more complex than an object A in the same category if there exists a monomorphism from A to B. (For a review on this and other categorical concepts, see my tierce '*Categorical System Theory*' [6], or a standard reference on category theory such as [7].) For a simpler example, one may say that a set B is more complex (*i.e.*, more complicated) than a set A if A is a subset of B, from the mere fact that the set B may possibly contain elements that are not found in the set A.

Next, in Sect. 5.7 and 7.1, respectively, of [4], one finds this:

- '…*we are going to define a system to be complex to the extent that we can observe it in non-equivalent ways.*'
- '…*this category of general dynamical systems, in which all science has hitherto been done, is only able to represent what I call* simple systems *or* mechanisms. *Natural systems which have mathematical images lying outside of this category and which accordingly do not admit a once-and-for-all partition into states plus dynamical laws, are thus not simple systems; they are* complex.'

The final refinement of the definitions of simple and complex systems appears in Chapt. 19 of [2], and I state it formally as:

Definition 2.1. A system is *simple* if all of its models are simulable. A system that is not simple, and that accordingly must have a nonsimulable model, is *complex*.

A *model* is a commutative encoding and decoding between two systems in what *Rosen* calls a *modeling relation* (for a review, see Chapts. 2 and 3 of [4]). A model is *simulable* if it is '*definable by an algorithm*' (see Chapt. 7 of [5]). It is variously called '*computable*', '*effective*', and '*evaluable by a mathematical (*Turing*) machine*'. The characterization of simulability applies to formal systems. A *formal system* in *Rosen*'s lexicon simply means '*an object in the universe of mathematics*'. It includes, but is not limited and, therefore, not equivocated to *Hilbert*'s formalism. In this context, a simple system according to *Rosen* is a natural system with the property that every formal system that encodes it through the modeling relation is simulable.

In each refinement of the definitions, the simple system/complex system dichotomy is preserved. This dichotomy, indeed, is essentially what distinguishes the '*absolute*' complexity of *Rosen* from the '*relative*' complexity of '*other authors*'. *Rosen* wrote in Chapt. 2 of [2]:

'*A system is complex if it has noncomputable models. This characterization has nothing to do with more complication, or with counting of parts or interactions; such notions, being themselves predicative, are beside the point.*

It should be noted that there are many deep parallels between the dichotomy we have drawn between simple (predicative) and complex (impredicative), and the dichotomy between the finite and the infinite. Just as 'infinite' is not just 'big finite', impredicatives are not just big (complicated) predicatives. In both cases, there is no threshold to cross, in terms of how many repetitions of a rote operation such as 'add one' are required to carry one from one realm to the other, nor yet back again.'

In the introduction of [1] quoted in the previous chapter, and in the informal definition of Sect. 7.1 of [4], as quoted above, the term *mechanism* is introduced as a synonym of *simple system*. The same equivalence also appears in Sect. 7B of [5]:

'*I shall express these notions in terms of true modelling relations and shall be led thereby to a new class of systems, which I shall call* simple systems *or* mechanisms. *These are characterized by the property that every model of them is simulable.*'

The formal definition of *mechanism* then appears in Sect. 8B of [5], and I state it formally as:

Definition 2.2. A natural system *N* is a *mechanism* if and only if all of its models are simulable.

The definition of *mechanism* is identical to the above definition (*Definition 2.1*) of the term *simple system*. So far, I have presented the *Rosen* etymology of *mechanism* = *simple system* and of *complex system*. The fourth term, *organism*, will have to wait, until I introduce a little mathematical logic.

3. Conditional Statements and Variations. – *Lewis Carroll* is the pen-name of the logician *Rev. Charles Lutwidge Dodgson*. Here is another episode in his Alice's adventures, this time from Chapt. VII of '*Alice's Adventures in Wonderland*' (1865):

> '*Then you should say what you mean', the March Hare went on.*
> *'I do', Alice hastily replied; 'at least – at least I mean what I say – that's the same thing, you know'.*
> *'Not the same thing a bit!', said the Hatter. 'Why, you might just as well say that «I see what I eat» is the same thing as «I eat what I see»!'*
> *'You might just as well say', added the March Hare, 'that «I like what I get» is the same thing as «I get what I like»!'*
> *'You might just as well say', added the Dormouse, which seemed to be talking in his sleep, 'that «I breathe when I sleep» is the same thing as «I sleep when I breathe»!'*
> *'It is the same thing with you', said the Hatter, and here the conversation dropped, and the party sat silent for a minute,…'*

Many statements, especially in mathematics, are of the form: 'if p, then q'. These are called *conditional statements*, and are denoted in the predicate calculus of formal logic by:

$$p \rightarrow q \tag{1}$$

The 'if clause' p is called the *antecedent*, and the 'then clause' q is called the *consequent*. Note that the conditional form *1* may be translated equivalently as '*q if p*'. So, the clauses of the sentence may be written in the reverse order, when the antecedent does not in fact 'go before', and the conjunction 'then' does not explicitly appear in front of the consequent.

If the antecedent is true, then the conditional statement is true if the consequent is true, and the conditional statement is false if the consequent is false. If the antecedent is false, then the conditional statement is true, regardless of whether the consequent is true or false. In other words, the conditional $p \rightarrow q$ is false if p is true and q is false, and it is true otherwise.

Alice's '*I do*' is the contention '*I say what I mean*'. This may be put as the conditional statement: 'If I mean it, then I say it'. This corresponds to form *1*, with p = 'I mean it', and with q = 'I say it'. It is equivalent to the statement: 'I say it if I mean it'.

The conditional form *1* may also be read as '*p only if q*'. *Alice*'s statement is then: '*I mean it only if I say it*'. The adverb 'only' has many nuances, and in common usage, 'only if' is sometimes used simply as an emphasis of 'if'. But in mathematical logic, 'only if' means 'exclusively if'. So, 'p only if q' means: 'if q does not hold, then p cannot hold either'. In other words, it is logically equivalent to 'If not q, then not p', which in the predicate calculus is:

$$\neg q \rightarrow \neg p \tag{2}$$

where the symbol '¬' denotes *negation* (the logical *not*). The conditional form *2* is called the *contrapositive* of form *1*. The contrapositive of *Alice*'s '*I mean it only if I say it*' (*i.e.*, '*if I mean it, then I say it*') is the equivalent conditional statement: '*If I do not say it, then I do not mean it*'.

The conditional form

$$q \rightarrow p \tag{3}$$

is called the *converse* of the form *1*, and the equivalent contrapositive of the converse, *i.e.*, the conditional form

$$\neg p \rightarrow \neg q \tag{4}$$

is called the *inverse* of the original form *1*. A conditional statement and its converse or inverse are *not* logically equivalent. For example, if p is true and q is false, then the conditional $p \rightarrow q$ is false, but its converse $q \rightarrow p$ is true. The confusion between a conditional statement and its converse is a common mistake. *Alice* thought '*I mean what I say*' (*i.e.*, the converse statement '*If I say it, then I mean it*') was the same thing as '*I say what I mean*' (the original conditional statement '*if I mean it, then I say it*'), and was then thoroughly ridiculed by her Wonderland acquaintances.

The conjunction

$$(p \rightarrow q) \wedge (q \rightarrow p) \tag{5}$$

(where the symbol '∧' is the logical *and*) is abbreviated into

$$p \leftrightarrow q, \tag{6}$$

called a *biconditional* statement. Since $q \rightarrow p$ may be read 'p if q', and since $p \rightarrow q$ may be read 'p only if q', the biconditional statement is 'p *if and only if* q', which often is abbreviated as 'p *iff* q'. If p and q have the same truth value (*i.e.*, either both are true or both are false), then the biconditional statement $p \leftrightarrow q$ is true; if p and q have opposite truth values, then $p \leftrightarrow q$ is false.

4. Implications. – In mathematics, theorems (also propositions, lemmata, and corollaries) assert the truth of statements. Grammatically speaking, they should have as their subjects the statement (or the name of, or some other reference to, the statement), and as predicates the phrase 'is true' (or 'holds', or some similar such). For example, the concluding *Rosen* theorem in Sect. 9G of [5] is:

Theorem 4.1. There can be no closed path of efficient causation in a mechanism.

This should be understood as:

Theorem 4.1′. 'There can be no closed path of efficient causation in a mechanism' is true.

Or, what is the same:

Theorem 4.1′. Theorem 4.1 is true.

But, of course, this *Theorem 4.1′* really means:

Theorem 4.1″. *Theorem 4.1′ is true.*

Or, equivalently:

Theorem 4.1″. 'Theorem 4.1 is true' is true.

This idea of a 'statement about a statement' may, alas, be iterated *ad infinitum*, to:

Theorem 4.1$^{\omega}$. …''''Theorem 4.1 is true' is true' is true' is true' is true…

Lewis Carroll wrote about this hierarchical 'Reasoning about Reasoning' paradox in a witty dialogue '*What the Tortoise said to Achilles*' [8]. Efficiency and pragmatism dictate the common practice that the predicate is implicitly assumed, and hence usually omitted. A theorem, then, generally consists of just the statement itself, the truth of which it asserts.

An *implication* is a *true* statement of the form:

$$\text{`}p \rightarrow q\text{' is true} \tag{7}$$

It is a statement about (the truth of) the conditional statement:

$$\text{`}p \rightarrow q\text{'} \tag{8}$$

The implication *7* is denoted in formal logic by

$$p \Rightarrow q, \tag{9}$$

which is read as '*p implies q*'. When a conditional statement is expressed as a theorem in mathematics, *viz.*

Theorem. If p, then q.

it is understood in the sense of *7*, that it is an implication.

The difference between the symbols '$\rightarrow$' and '$\Rightarrow$', *i.e.*, between a conditional statement and an implication, is that of syntax and semantics. Note that $p \rightarrow q$ is just a proposition in the predicate calculus, which may be true or false. But the form $p \Rightarrow q$ is a statement *about* the conditional statement $p \rightarrow q$, asserting that the latter is a true statement. In particular, when $p \Rightarrow q$, the situation that p is true and q is false (which is the only circumstance for which the conditional $p \rightarrow q$ is false) *cannot* occur.

Since a conditional statement and its contrapositive are equivalent, when $p \rightarrow q$ is true, the form $\neg q \rightarrow \neg p$ is also true. The contrapositive inference

$$(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p) \tag{10}$$

is itself an implication, called *modus tollens* in mathematical logic.

Most mathematical theorems are stated, or may be rewritten, as implications. *Rosen*'s *Theorem 4.1*, *e.g.*, is $p \Rightarrow q$, with p = 'N is a mechanism', and with q = 'there is no closed path of efficient causation in N', where N is a natural system. Stated explicitly, it is the following theorem:

Theorem 4.2. If a natural system N is a mechanism, then there is no closed path of efficient causation in N.

The equivalent contrapositive implication $\neg q \Rightarrow \neg p$ then leads to:

Theorem 4.3. If a closed path of efficient causation exists in a natural system N, then N cannot be a mechanism.

A true statement of the form:

$$\text{`}p \leftrightarrow q\text{' is true,} \tag{11}$$

which asserts the truth of a biconditional statement, is called an *equivalence*. It is denoted as:

$$p \Leftrightarrow q \tag{12}$$

and is read as 'p and q are *equivalent*'. It is clear from the definitions that the equivalence *12* is equivalent to the conjunction:

$$p \Rightarrow q \text{ and } q \Rightarrow p \tag{13}$$

When $p \Leftrightarrow q$, either both p and q are true, or both are false. When a biconditional statement is expressed as a theorem in mathematics, *viz.*

Theorem. p if and only if q.

it is understood in the sense of *11* that the biconditional statement $p \leftrightarrow q$ is, in fact, true, and that it is the equivalence $p \Leftrightarrow q$.

A *definition* is trivially a theorem – by definition, as it were. It is also often expressed as an equivalence, *i.e.*, with an 'if and only if' statement. See, *e.g.*, the above *Definition 2.2* of 'mechanism'.

Occasionally, a definition may be *stated* as an implication (see, *e.g.*, *Definition 2.1* of 'simple system'), but in such cases the converse is *implied* (by convention, or, indeed, by definition). Stated otherwise, a definition is always an equivalence, whether it is expressed as such or not, between the term being defined and the defining conditions. As a simple example, consider the definition of 'subset', which may be given as 'A set A is a subset of a set B if every element of A is an element of B.' This is the implication $p \Rightarrow q$, where p = 'every element of A is an element of B', and q = 'set A is a subset of set B'. But since this is a definition, implicitly entailed is the converse $q \Rightarrow p$: 'If a set A is a subset of a set B, then every element of A is an element of B.' So the definition is really:

Definition 4.4. A set A is a *subset* of a set B *if and only if* every element of A is an element of B.

Note that this implicit entailment is *not* a contradiction to the fact, discussed in the previous chapter, that a conditional statement is not logically equivalent to its converse. The propositions $p \rightarrow q$ and $q \rightarrow p$ will always remain logically distinct, and in general the implication $p \Rightarrow q$ says nothing about $q \Rightarrow p$. The previous paragraph only applies to definitions, and its syntax is:

$$\text{'If } p \Rightarrow q \text{ is a definition, then also } q \Rightarrow p\text{ , whence } p \Leftrightarrow q\text{'} \tag{14}$$

5. Necessity and Sufficiency. – The law of inference

$$\text{'If } p \Rightarrow q \text{ and } p \text{ is true, then } q \text{ is true'} \tag{15}$$

is called *modus ponens*. This inference follows from the fact that when $p \Rightarrow q$, $p \rightarrow q$ is true; so the situation that p is true and q is false (the only circumstance for which the conditional $p \rightarrow q$ is false) cannot occur. Thus, the truth of p predicates q. Incidentally, *modus ponens* is the 'theorem' that begins the propositional canon in [8]. Note that the truth of $p \rightarrow q$ is required for the truth of p to entail the truth of q. In a general (not necessarily true) conditional statement $p \rightarrow q$, the truth values of p and q are independent.

Because of its inferential entailment structure (that the truth of p is sufficient to establish the truth of q), the implication $p \Rightarrow q$ may also be read as 'p is *sufficient* for q'. Contrapositively (hence equivalently), the falsehood of q is sufficient to establish the falsehood of p. In other words, if q is false, then p cannot possibly be true; *i.e.*, the truth of q is necessary (although some additional true statements may be required) to establish the truth of p. Thus, the implication $p \Rightarrow q$ may also be read as 'q is *necessary* for p'. The equivalence $p \Leftrightarrow q$ (*i.e.*, when 'p iff q' is true so that p and q predicate each other) may, therefore, be read as '*p is necessary and sufficient for q*'.

The concepts of necessity and sufficiency are intimately related to the concept of subset. *Definition 4.4* is the statement:

$$A \subset B \text{ iff } \forall x\ (\mathrm{x} \in A) \Rightarrow (\mathrm{x} \in B) \tag{16}$$

Stated otherwise, when A is a subset of B (or, what is the same, B *includes* A), membership in A is *sufficient* for membership in B, and membership in B is *necessary* for membership in A.

The most basic property in set theory is that of *equality*, which is formulated as the following axiom:

Axiom of Extension. Two sets are equal if and only if they have the same elements.

If A and B are sets such that $A \subset B$ and $B \subset A$ (*i.e.*, membership in A and in B are necessary and sufficient for each other), then the two sets have the same elements. The converse is equally obvious. The axiom of extension may, then, be restated as:

Theorem. Two sets A and B are equal iff $A \subset B$ and $B \subset A$.

On account of this theorem, the proof of set equality $A = B$ is usually split into two parts: first prove that $A \subset B$, and then prove that $B \subset A$.

The major principle of set theory is the following axiom:

Axiom of Specification. For any set U and any statement $p(x)$ about x, there exists a set P the elements of which are exactly those $x \in U$ for which $p(x)$ is true.

It follows immediately from this axiom that the set P is determined uniquely. To indicate the way P is obtained from U and $p(x)$, the customary notation is:

$$P = \{x \in U : p(x)\} \tag{17}$$

The term '$p(x)$' in form *17* is understood to mean "$p(x)$' is true' (with the conventional omission of the predicate); it may also be read as 'x has the property p'. For example, let $\mathbf{N}$ be the set of all natural systems, and let $s(N) =$ 'all models of N are simulable'. Then, one may denote the set of all simple systems $\mathbf{S}$ (*cf. Definition 2.1*) (and synonymously the set of all mechanisms $\mathbf{M}$ (*cf. Definition 2.2*)) as:

$$\mathbf{S} = \mathbf{M} = \{N \in \mathbf{N} : s(N)\} \tag{18}$$

When the 'universal set' U is obvious from the context (or inconsequential), it may be dropped, and the notation *17* abbreviates to:

$$P = \{x : p(x)\} \tag{19}$$

As a trivial example, a set A may be represented as:

$$A = \{x : x \in A\} \tag{20}$$

Statement *16* connects set inclusion with implication of the membership property. Analogously, if one property implies another, then the set specified by the former is a subset of the set specified by the latter (and conversely). Explicitly, if 'x has the property p' implies that 'x has the property q', *i.e.*, if $\forall x\, p(x) \Rightarrow q(x)$, then $P = \{x : p(x)\}$ is a subset of $Q = \{x : q(x)\}$ (and conversely):

$$P \subset Q \text{ iff } \forall x\ (p)x \Rightarrow q(x) \tag{21}$$

For example, let $\mathbf{N}$ be the set of all natural systems, with $t(N) =$ 'there is no closed path of efficient causation in N', and let:

$$\mathbf{T} = \{N \in \mathbf{N} : t(N)\} \tag{22}$$

Let **M** be the set of all mechanisms as specified in form *18* by $s(N)$ = 'all models of N are simulable'. *Theorem 4.2* (the proof of which is the content of Chapt. 9 of [5]) is the statement:

$$\forall N \in \mathbf{N}\ s(N) \Rightarrow t(N) \tag{23}$$

whence

$$\mathbf{M} \subset \mathbf{T} \tag{24}$$

On the other hand, it is clear that if a natural system has no closed path of efficient causation, then none of its models can have hierarchical cycles, so they are all fractionable, whence simulable. (This is also discussed in detail as part of the relational theory of machines in Chapt. 9 of [5].) Thus, while implication *23* itself requires all of Chapt. 9 to prove (after establishing some 'conclusions' in Chapt. 8 of [5]), trivially true is its converse:

$$\forall N \in \mathbf{N}\ t(N) \Rightarrow s(N) \tag{25}$$

So, one also has:

$$\mathbf{T} \subset \mathbf{M}; \tag{26}$$

and, together with inclusion *24*, this means:

$$\mathbf{M} = \mathbf{T} \tag{27}$$

Therefore, we can state the following theorem:

Theorem 5.1. A material system is a mechanism if and only if it has no closed path of efficient causation.

6. Complements. – The *relative complement* of a set A in a set B is the set of elements in B, but not in A:

$$B \sim A = \{x \in B : x \notin A\} \tag{28}$$

When B is the 'universal set' U (of some appropriate universe under study, *e.g.*, the set of all natural systems **N**), the set $U \sim A$ is denoted as A^c, *i.e.*:

$$A^c = \{x \in U : x \notin A\}, \tag{29}$$

and is called simply the *complement* of the set A. An element of U is either a member of A, or not a member of A, but not both. That is, $A \cup A^c = U$, and $A \cap A^c = \varnothing$.

The set specified by the property p, *i.e.*, $P = \{x : p(x)\}$, has as its complement the set specified by the property $\neg p$; *i.e.:*

$$P^c = \{x : \neg p(x)\} \tag{30}$$

The negation of the statement $s(N) =$ 'all models of N are simulable' is: $\neg s(N) =$ 'there exists a model of N that is not simulable'. This, by the way, is a *law of quantifier negation* in the predicate calculus:

$$\neg \forall x\ p(x) \Leftrightarrow \exists x\ \neg p(x) \tag{31}$$

Thus, the set of all complex systems **C** is, by definition, the complement of the set of simple systems **S**:

$$\mathbf{C} = \mathbf{S}^c = \{N \in \mathbf{N} : \neg s(N)\} \tag{32}$$

Since $\mathbf{S} = \mathbf{M}$ and $\mathbf{M} = \mathbf{T}$ (*Eqns. 18* and *27*, resp.), we can write:

$$\mathbf{C} = \mathbf{M}^c = \mathbf{T}^c \tag{33}$$

The negation of the statement $t(N) =$ 'there is no closed path of efficient causation in N' is $\neg t(N) =$ 'there exists a closed path of efficient causation in N'. Any natural system, through the modeling relation, generally has many different models that do not contain any impredicative structures (or hierarchical cycles) of inferential entailment. The limitation for a mechanism (or a simple system) is that this impoverishment applies to *all* of its models (*Theorem 5.1*). The defining property of a complex system is that there exists at least one model that does contain an impredicative structure of inferential entailment, one that would correspond to the closed path of efficient causation in the complex system being modeled. The complementarity

$$\mathbf{C} = \mathbf{T}^c = \{N \in \mathbf{N} : \neg t(N)\} \tag{34}$$

then leads to the next theorem:

Theorem 6.1. A natural system is *complex* if and only if it contains a closed path of efficient causation.

Chapt. 18 of [2] contains a detailed discussion of *Theorem 6.1* and its consequences.

7. Organism. – After the interlude in mathematical logic, we are now properly equipped to study the fourth term: *organism*. In Chapt. 10 of [5], *Rosen* proposed an answer to the ultimate biological question of 'what is life?':

Definition 7.1. A material system is an *organism* if and only if it is closed to efficient causation.

A natural system is said to be *closed to efficient causation* if its every efficient cause is entailed within the system. In other words, the definition of 'closure to efficient causation' for a natural system is that it has a corresponding formal system model that has a closed path containing *all* of the representations of efficient causes in its causal entailment structure. Note that a complex system is *not* necessarily closed to efficient causation. A complex system, according to *Theorem 6.1*, is only required to have *a* cycle containing *some*, but not necessarily all, efficient causes; on the other hand, an organism, by *Definition 7.1*, has a cycle that contains them *all*. A complex system is 'not a mechanism'; a living system requires a little bit more. This is how one distinguishes between complex and living systems.

Stated otherwise, an organism must be complex; but a complex system may (or may not) be an organism; *i.e.:*

$$\mathbf{O} \subset \mathbf{C}, \text{ but } \mathbf{C} \not\subset \mathbf{O} \tag{35}$$

whence:

$$\mathbf{O} \neq \mathbf{C} \tag{36}$$

Among the four terms that I set out to study in this paper, 'simple system' and 'complex system' are not 'everyday terms', have no 'standard definitions', and hence may be used according to *Rosen*'s *Definition 2.1* without too often encountering the fallacy of semantic equivocation. It may also be argued that the *relational Definition 2.2* of 'mechanism' agrees well with its common kinematic *structural* definition, for example, as in 'an assemblage of bodies formed and connected to move upon one another with definite relative motion'. But the word 'organism' has a well-established meaning in common usage, that of 'autonomous life form'. It is notable that in the passage from [1], as quoted in *Chapt. 1*, *Rosen* italicized the first three terms (indicating that he had his own technical definitions for them), but did not for the word 'organism' (which implied that it was to be interpreted with its everyday meaning).

Since words mean 'neither more nor less' than what they are defined to mean, it would be perfectly logically consistent if one just takes *Rosen*'s *Definition 7.1* as *the definition* of the word 'organism' and proceeds. But 'reality', alas, intervenes. If an organism has to reflect the 'reality' of *life*, further explanation of *Definition 7.1* is in order.

8. (*M*,*R*)-Systems. – In the final, concluding Section 11 H of [5], *Rosen* wrote: '*But complexity, though I suggest it is the habitat of life, is not itself life. Something else is needed to characterize what is alive from what is complex.*' And in Chapt. 1 of [2], after explaining that a living system must have nonsimulable models, hence must be complex, he added: '*To be sure, what I have been describing are necessary conditions, not sufficient ones, for a material system to be an organism. That is, they really pertain to what is not an organism, to what life is not. Sufficient conditions are harder; indeed, perhaps there are none. If so, biology itself is more comprehensive than we presently know.*'

Rosen established the necessity for life with his detailed exposition in [5]. He chose to define the sufficiency, using a class of relational models of organisms called (*M*,*R*)-systems. The comprehensive reference is [9]. *Rosen* also discussed them in Sect. 3.5 of [4], in Sect. IV and V of [1], in Sect. 10C of [5], and in Chapt. 17 of [2]. I have written on their noncomputability and realizations in two recent papers [10][11]. The reader may refer to any or all of the above for their details, which need not be repeated here.

For our present purpose, we only need to know that an (M,R)-system is an example of a formal model of a natural system that is closed to efficient causation. In other words, all its efficient causes are contained in an impredicative cycle of inferential entailment, and it is, therefore, nonsimulable. By *Definition 7.1*, an (M,R)-system is a model of an organism. The inclusion $\mathbf{O} \subset \mathbf{C}$ means that any natural system that realizes an (M,R)-system must be complex. Note, however, the implication (hence inclusion) only goes one way; the converse is not true. An (M,R)-system has a very special relational organization: in particular, it is closed to efficient causation, having *all* its efficient causes in a cycle. A complex system is only required to contain *a* cycle containing *some* of its efficient causes (whence entailing its nonsimulability): there may, however, be efficient causes that are not part of the cycle. So, not every nonsimulable formal system is an (M,R)-system, whence not every complex system has an (M,R)-system model.

Rosen's idea behind (M,R)-systems was, as he explained in Chapt. 17 of [2], '*to characterize the minimal organization a material system would have to manifest or realize to justify in calling it a* cell'. His definition of a cell is:

Definition 8.1. A *cell* is (at least) a material structure that realizes an (M,R)-system.

The word 'cell' is used here also in the sense of 'autonomous life form', *i.e.*, 'organism'. This definition, thus, says that 'having an (M,R)-system as a model' is a necessary condition for a natural system to be an autonomous life form. Immediately after *Definition 8.1*, in the same paragraph in Chapt. 17 of [2], *Rosen*, in fact, added: '*Making a cell means constructing such a realization. Conversely, I see no grounds for refusing to call such a realization an autonomous life form, whatever its material basis may be.*' The converse statement provides the sufficiency. So an alternative definition of 'organism' is:

Definition 8.2. A natural system is an *organism* if and only if it realizes an (M,R)-system.

Definitions 8.2 and *7.1* are consistent. 'Having an (M,R)-system as a model', or, equivalently, 'closed to efficient causation', is the necessary and sufficient condition for a natural system to be an autonomous life form, on a relational level, even if one may not readily recognize the natural system as 'alive' on a material level. This is the definition of 'organism' in the *Rosen* canon. And this is the definition of 'organism' in relational biology.

Let $r(N)$ = 'N has an (M,R)-system as its model'. Then the set of all organisms is:

$$\mathbf{O} = \{N \in \mathbf{N} : r(N)\} \tag{37}$$

With this definition of $\mathbf{O}$, the conditions *35* and *36*, $\mathbf{O} \subset \mathbf{C}$, but $\mathbf{C} \not\subset \mathbf{O}$ (whence $\mathbf{O} \neq \mathbf{C}$), are satisfied. The set of organisms is a *proper subset* of the set of complex systems.

9. The New Taxonomy. – We set out in this essay to study the following four subsets of the set **N** of natural systems:

$$\mathbf{S} = \{N \in \mathbf{N} : N \text{ is a } \textit{simple system}\}$$

$$\mathbf{M} = \{N \in \mathbf{N} : N \text{ is a } \textit{mechanism}\}$$

$$\mathbf{C} = \{N \in \mathbf{N} : N \text{ is a } \textit{complex system}\}$$

$$\mathbf{O} = \{N \in \mathbf{N} : N \text{ is an } \textit{organism}\}$$

We discovered that, in the *Rosen* etymology, these are their relations:

$$\mathbf{C} = \mathbf{S}^c \tag{38}$$

$$\mathbf{S} = \mathbf{M} \tag{39}$$

$$\mathbf{O} \subset \mathbf{C} \text{ (but } \mathbf{O} \neq \mathbf{C}) \tag{40}$$

I will now briefly discuss two other related concepts, *anticipatory system* and *machine*, and see where they fit in this relational universe. *Rosen*'s definitive and pioneering treatment of *anticipatory systems* is, of course, given in the monograph [4]. The reader is referred to the original text for the details in their full glory. Here is the basic definition:

> **Definition 9.1.** An *anticipatory system* is a natural system that contains an internal predictive model of itself and of its environment, which allows it to change state at an instant in accord with the model's predictions pertaining to a later instant.

In Sect. 6.8 of [4], *Rosen* summarized thus: '*The point of departure for our entire development was the recognition that most of the behavior we observe in the biological realm, if indeed not all of the behavior which we consider as characteristically biological, is of an anticipatory rather than a reactive character. In fact, if it were necessary to try to characterize in a few words the difference between living organisms and inorganic systems, such a characterization would not involve the presence of DNA, or any other purely structural attributes; but rather that organism constitute the class of systems which can behave in an anticipatory fashion. That is to say, organisms comprise those systems which can make predictive models (of themselves, and of their environments) and use these models to direct their present actions.*'

In short, anticipation is a necessary condition for life. Let $\mathbf{A} = \{N \in \mathbf{N} : N \text{ is an } \textit{anticipatory system}\}$, then:

$$\mathbf{O} \subset \mathbf{A}, \text{ but } \mathbf{A} \not\subset \mathbf{O} \text{ (whence } \mathbf{A} \neq \mathbf{O}) \tag{41}$$

Rosen concluded in the final paragraph of Chapt. 7 of [4]: '*Our final conceptual remark is also in order. As we pointed out above, the Newtonian paradigm has no room for the* *category of final causation. This category is closely tied up with the notion of anticipation, and in its turn, with the ability of systems to possess internal predictive models of themselves and their environments, which can be utilized for the control of present actions. We have argued at great length above that anticipatory control is indeed a distinguishing feature of the organic world, and developed some of the unique features of such anticipatory systems. In the present discussion, we have in effect shown that, in order for a system to be anticipatory, it must be complex. Thus, our entire treatment of anticipatory systems becomes a corollary of complexity. In other words, complex systems can admit the category of final causation in a perfectly rigorous, scientifically acceptable way. Perhaps this alone is sufficient recompense for abandoning the comforting confines of the Newtonian paradigm, which has served us so well over the centuries. It will continue to serve us well, provided that we recognize its restrictions and limitations as well as its strengths.*'

The corollary is: an anticipatory system must be complex; a complex system may (or may not) be anticipatory. In other words:

$$\mathbf{A} \subset \mathbf{C}, \text{ but } \mathbf{C} \not\subset \mathbf{A} \text{ (whence } \mathbf{A} \neq \mathbf{C}) \tag{42}$$

Thus, one has, in view of relations *41* and *42*, the hierarchy:

$$\mathbf{O} \subset \mathbf{A} \subset \mathbf{C} \tag{43}$$

in which both inclusions are proper.

The formal definition of *machine* appears in Section 8B of [5], immediately following that of *mechanism:*

> **Definition 9.2.** A natural system is a *machine* if and only if it is a mechanism, such that at least one of its models is already a mathematical machine.

Let $\mathbf{K} = \{N \in \mathbf{N} : N \text{ is a } machine\}$; then, by definition, we have the inclusion:

$$\mathbf{K} \subset \mathbf{M} \tag{44}$$

The subtle difference between **K** and **M** is as follows. For a mechanism $N \in \mathbf{M}$, every model is simulable. This means every model may be *simulated* by a mathematical (*Turing*) machine. For a machine $N \in \mathbf{K}$, all of its models are simulable, *and* among them, at least one must be *modeled* by a mathematical machine. See Chapt. 7 in [5] for a detailed discussion of simulations and models. In other words, 'simulable' means 'has a mathematical machine simulation'; and 'is a mathematical machine' means 'has a mathematical machine model'. (The modeling relation is a natural isomorphism, and mathematicians identify isomorphic objects.) That is, for a mechanism to be a machine, one of those simulations must be more than that: it must be a model, in the sense discussed in Sect. 7F of [5][1]). The distinction between mechanisms and machines is thus:

Definition 2.2′. A natural system is a *mechanism* if and only if every one of its models has a mathematical machine simulation.

Definition 9.2′. A natural system is a *machine* if and only if it is a mechanism, such that at least one of its models has a mathematical machine model.

So, there are mechanisms that are not machines, whence $\mathbf{K} \neq \mathbf{M}$, and the inclusion *44* is also a proper one.

In Chapt. 19 of [2], *Rosen* used the diagram shown in *Fig. 1* (there 'figure 19.2') to represent the world of *von Neumann* complexity: the universality of physics, the machine metaphor, and the finite threshold of complexity.

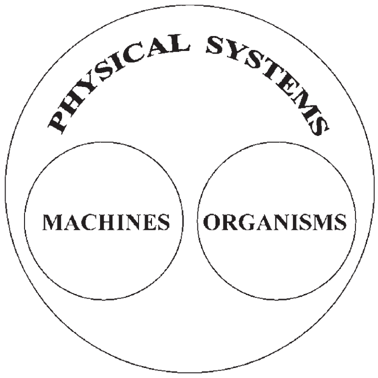

Fig. 1. *The world of* von Neumann *complexity according to* Robert Rosen (taken from Chapt. 19 of [2], there figure 19.2)

He then suggested a taxonomy for natural systems that is profoundly different from the concept shown in *Fig. 1*: '*The nature of science itself (and the character of*

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[1]) While we are on the topic of simulations and models in Sect. 7F of [5], it is appropriate to mention that the entailment structure of a model was misprinted in [5]. It should read

$$\alpha(\varphi) \Rightarrow (\alpha(x) \Rightarrow \alpha(\varphi(x))). \qquad [7F.2]$$

And the 'completely different' entailment structure of simulation may be more clearly represented as

$$\psi \Rightarrow ([\alpha(\varphi), \alpha(x)] \Rightarrow \alpha(\varphi(x))). \qquad [7F.3]$$

These two arrow diagrams contrast succinctly the difference between model and simulation. A simulation of a process provides an alternate description of the *entailed effects*, whereas a model additionally also provides an alternate description of the *entailment structure of the mapping* representing the process itself.

technologies based on sciences) depends heavily on whether the world is like figure 19.2 or like this new taxonomy.

In this new taxonomy, there is a partition between mechanisms and nonmechanisms. Let us compare its complexity threshold with that of figure 19.2. In figure 19.2, the threshold is porous; it can be crossed from either direction, *by simply repeating a single rote (syntactic) operation sufficiently often – an operation that amounts to 'add one' (which will ultimately take us from simple to complex) or 'subtract one' (which will ultimately take us from complex to simple).*

In the new taxonomy, on the other hand, the barrier between simple and complex is not porous; it cannot be crossed at all in the direction from simple to complex; even the opposite direction is difficult. There are certainly no purely syntactic operations that will take us across the barrier at all. That is, no given finite number of repetitions of a single rote operation will take us across the barrier in either direction; it can produce neither simplicity from complexity, nor the reverse.'

In *Fig. 2*, I have drawn my own diagram of this new taxonomy, showing all the subsets of natural systems discussed in this essay. This is the succinct summary of *Rosen*'s science.

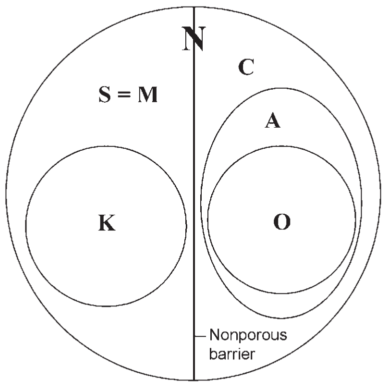

Fig. 2. *Completed scheme of* Rosen*'s taxonomy*

I defer the final word on the subject to *Robert Rosen*, himself, taken from the introduction to Part V of [2]: '*I am always asked by experimentalists why I do not propose explicit experiments for them to perform, and subject my approaches to verification at their hands. I do not do so because, in my view, the basic questions of biology are not empirical questions at all, but, rather, conceptual ones; I tried to indicate this viewpoint in* Life Itself. *But the chapters in this part, I hope, expound the true empirical correlates of biological theory. In the realm of art and craft, rather than in a traditional laboratory, will ample verification be found.*'

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