

Anticipation in (M,R)-systems

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(Received 12 June 2011; final version received 12 July 2011)

Relational biology is a study of life in terms of the organization of entailment relations in living systems, independent of any particular physical mechanism or material realization. Anticipatory systems and metabolism–repair (M,R)-systems are two classes of relational models that characterize life. In this paper, I explore the connections between them: I explicate the circumstances under which anticipation occurs in an (M,R)-system, and show that the entailment pattern of an anticipatory system may be represented as a network of metabolism and repair components.

Keywords: anticipatory system; (M,R)-system; relational biology; hierarchical cycle; clef system; impredicativity

1. Objects

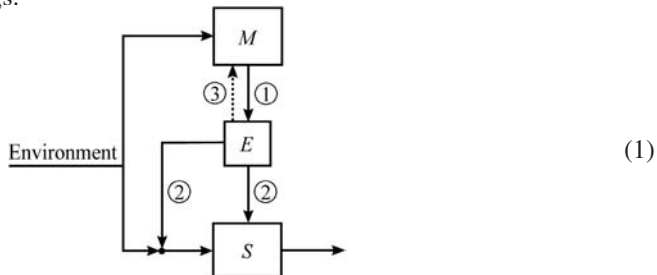
The objects of this paper are two classes of relational models that characterize life: anticipatory systems and metabolism–repair (M,R)-systems. Both were introduced by the mathematical biologist Robert Rosen (1934–1998).

1.1 Anticipatory system

Rosen first used the term ‘anticipatory system’ in the paper ‘Planning, management, policies and strategies: four fuzzy concepts’ (Rosen 1974¹), published in the first volume of this journal. Thence, he defined the ‘anticipatory modes of behaviour of organisms’ to be those:

in which an organism’s present behaviour is determined by: (a) sensory information about the present state of the environment; and (b) an ‘internal model’ of the world, which makes predictions about future states on the basis of the present data and the organism’s possible reactions to it.

It was also in this paper that the now-iconic diagram of an anticipatory system first appeared in Rosen’s writings:



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In diagram (1), *S*, *M*, and *E* are, respectively, object system, predictive model, and set of effectors. (I shall in later sections return to this diagrammatic representation.)

Biology is replete with situations in which organisms can generate and maintain internal predictive models of themselves and their environments, and use the predictions of these models about the future for purpose of control in the present. This is true at every level, from the molecular to the cellular to the physiological to the behavioural, and this is true in all parts of the biosphere, from microbes to plants to animals to ecosystems. One may succinctly postulate the

Axiom of Anticipation: Life is anticipatory.

But anticipatory behaviour, while a necessary condition for life, is not restricted to the biological universe; it, indeed, encompasses the ‘four fuzzy concepts’, and more. At the human level it can be multiplied without end, and may seem fairly trivial: examples range from avoiding dangerous encounters to any strategy in games and sports.

Although the *concept* of ‘anticipation’ had not been new, the *systemic study* of it was when Rosen wrote his 1974 paper. Rosen’s rigorously mathematical study of this biology-inspired subject led to a sequence of papers, culminating in his book *Anticipatory Systems* (Rosen 1985). Therein is the generalized, formal definition as follows.

DEFINITION. An *anticipatory system* is a natural system that contains an internal predictive model of itself and of its environment, which allows it to change state at an instant in accord with the model’s predictions pertaining to a later instant.

An anticipatory system’s present behaviour depends upon ‘future states’ or ‘future inputs’ generated by an internal predictive model. ‘Model-based behaviour’ (or more specifically ‘anticipatory-model-based behaviour’) is the essence of social, economic, and political activity. Beyond its organismic origins, an anticipatory system finds analogies in social systems, economics, politics, ethics, future studies, and many others. The common question in these diverse fields is that of *ought*, which may be phrased as ‘What should we do now?’ However different the contexts in which the question is posed, *they are all alike in their fundamental concern with the making of policy, and the associated notions of forecasting the future and planning for it.* What is sought, in each of these diverse areas, is in effect a strategy of decision making. An understanding of the characteristics of model-based behaviour is thus central to any strategy that one wishes to develop to control and manage such systems, or to modify their model-based behaviour in new ways. But underlying any strategy there must be an underlying substratum of basic principles: a science, a *theory*. Rosen proposed that the theory underlying a strategy of policy generation is that of anticipatory systems.

Note, in contrast, that a *reactive system* can only react, in the present, to changes that have already occurred in the causal chain, while an *anticipatory system*’s present behaviour involves aspects of past, present, and future. The presence of a predictive model serves precisely to pull the future into the present; a system with a ‘good’ model thus behaves in many ways as if it can anticipate the future. In other words, *a predictive model permits anticipation.* Indeed, to use teleological language, the *purpose* of a predictive model is to anticipate. The ‘anticipatory paradigm’ *extends – but does not replace –* the ‘reactive paradigm’ which has dominated the study of natural systems, and allows us a glimpse of new and important aspects of system behaviour.

Relational biology is a study of life in terms of the organization of entailment relations in living systems, independent of any particular physical mechanism or material realization. It is a subject begun by Nicolas Rashevsky, and continued by his student Robert Rosen. For a thorough introduction to the subject, the enthused reader is invited to read my book *More than life itself: a synthetic continuation in relational biology* (Louie 2009). True to the spirit of relational biology, an anticipatory system is not defined using its material basis, but rather using a process that exists therein: that ‘function’ and not ‘structure’ is the important defining predicate.

1.2 (M,R)-system

Robert Rosen (1958) introduced (M,R)-systems to the world in 1958 in his very first published scientific paper. They began as a class of metaphorical, relational paradigms that define cells. M and R may very well stand for ‘metaphorical’ and ‘relational’ in modelling terms, but they are realized as ‘metabolism’ and ‘repair’. The comprehensive reference is Rosen (1972, and also Chapters 11–13 of Louie 2009). It may even be said that all of Rosen’s scientific work – his lifelong quest being the answer to the question ‘What is life?’ – has arisen from a consideration of topics related to the study of (M,R)-systems. This is because of the

Postulate of Life: A natural system is an *organism* if and only if it realizes an (M,R)-system.

Here, the word ‘organism’ is used in the sense of a general living system (including, in particular, cells). Thus, an (M,R)-system is the very model of life; and, conversely, life is the very manifestation of an (M,R)-system (*cf.* Chapter 11 of Louie).

Metabolism is represented by mappings of the form $f : A \rightarrow B$, whence its efficient cause, an *enzyme*, with material input and output represented by the sets A and B . In category-theoretic terms, metabolism is a morphism $f \in H(A, B) \subset B^A$ (where the hom-set $H(A, B)$ is a collection of mappings from set A to set B , and B^A is the collection of all mappings from A to B). Members of $H(A, B)$ are the mappings that model metabolic process, so clearly not all mappings in B^A qualify; thus, $H(A, B)$ is a proper subset of B^A .

Repair may be considered as mappings that create new copies of enzymes f , hence, under the one-gene–one-enzyme principle, *genes* that ‘repair’ the metabolism function. Thus, repair is a morphism Φ with codomain $H(A, B)$; i.e. $\Phi \in H(\cdot, H(A, B)) \subset H(A, B)$. Repair in cells generally takes the form of a continual synthesis of basic units of metabolic processor (i.e. enzymes), using as input materials provided by the metabolic activities themselves. In the simplest case, the domain of the repair map Φ is the codomain of a metabolism mapping f , its ‘output set’ B , whence $\Phi \in H(B, H(A, B)) \subset H(A, B)^B$.

The simplest (M,R)-system, with one each of metabolism and repair components, may therefore be represented by the diagram:

$$A \xrightarrow{f} B \xrightarrow{\Phi} H(A, B). \quad (2)$$

Metabolism corresponds to cellular activities that may collectively be called *cytoplasmic*, while repair corresponds to cellular activities that are *nuclear*. One of the decisive features of nuclear genetic activity in cells pertains to the *replication* of the genetic material. One purpose of replication is to replenish the repair component should the latter be damaged or otherwise diminished in efficiency. It is the genius of Rosen’s (M,R)-system formalism that, without the intervention of *ad hoc* assumptions, the

ingredients for such replication processes are already present. To replicate the repair components there are many ways, which I will not go into here. The reader is referred to Chapters 11–13 of Louie (2009) for the mathematical details. I shall, however, point out that the standard process of *semiconservative replication* of nucleic acid is but one of the many that can achieve the ultimate goal of repairing-the-repair in order to propagate the living system. In other words, replication in an (M,R)-system *may, but need not*, be realized as nucleic acid replication. It is also important to note that *replicability* is a *relational result*; i.e. independent of any particular physical mechanism or realization.

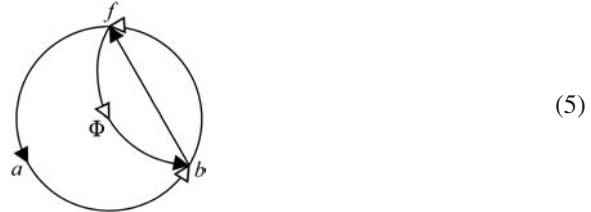
One way of extending the diagram of the simplest (M,R)-system (2) is:

$$A \xrightarrow{f} B \xrightarrow{\Phi} H(A, B) \xrightarrow{\beta} H(B, H(A, B)). \quad (3)$$

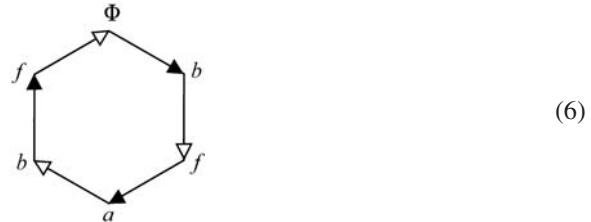
The magic of an (M,R)-system is that the *replication* mapping $\beta \in H(H(A, B), H(B, H(A, B)))$ may already be entailed in the original form (2). On the basis of what are already present in (2), Rosen shows that ‘under stringent but not prohibitively strong conditions, such replication essentially comes along for free’. An isomorphic identification between the sets B and $H(H(A, B), H(B, H(A, B)))$ (i.e. between $b \in B$ and β) is made – Rosen uses one due to the ‘inverse evaluation map’ with $\beta = \hat{b}^{-1}$. The mapping chain (3) in its element-chasing form is:

$$a \xrightarrow{f} b \xrightarrow{\Phi} f \xrightarrow{\beta} \Phi \quad (4)$$

(where, naturally, $a \in A$), which then folds into the following graphic representation of a cell, with the metabolism component as the abstract equivalent of ‘cytoplasm’ and the repair component as the abstract counterpart of ‘nucleus’:



In turn, graph (5) may be unfolded into the *hierarchical cycle*:



(The terminology and symbology will be explained in later sections.)

While all the ingredients are present within an (M,R)-system, replication is, nevertheless, not an *obligatory* feature of repair. Each entailment of replication from metabolism and repair depends on some ‘stringent but not prohibitively strong conditions’ imposed on the mapping(s) involved. This in turn depends on the character of the entire set $H(A, B)$ with which we are dealing. It may be expected that these conditions for ‘entailment closure’ will not usually be satisfied, and hence that most (M,R)-systems

cannot replicate. Thus, as we would expect, replication is a relatively rare and unusual situation.

Since not all (M,R)-networks satisfy the stringent requirements for entailment closure, those that do may acquire an adjective and be called *replicative (M,R)-systems*. A hierarchical cycle similar to (6) is the closure that provides the ‘self-sufficiency in efficient causes’ that defines replicative (M,R)-systems. The defining characteristic, in other words, is the self-sufficiency in the networks of metabolism–repair–replication components, in the sense that every mapping is entailed within; in short, *closure to efficient causation* (another term that I shall explicate below). Henceforth, I shall use the term *(M,R)-network* to describe a network of metabolism and repair components that is not necessarily closed to efficient causation. I shall drop the adjective ‘replicative’ for (M,R)-systems, whence all (M,R)-systems are replicative. (This is the sense in which the term appears in the Postulate of Life.) I will, however, postpone the formal definition until after the thorough discussion below on the defining properties.

Another noteworthy aspect of an (M,R)-system arises from the relation of the replication map to the other mappings involved. Note that the first two of the three maps in the chain (3) constitute our original (M,R)-system (2), in which f represents the metabolic component and Φ represents the repair component. But one may also see that the second two maps themselves constitute an (M,R)-system, except that now the original repair map Φ plays the role of the metabolic component, and the original replication map β plays the role of the repair component. From this, one sees the curious fact that *there is nothing intrinsic about the biological qualities of metabolism, repair, and replication*; our perception of them depends on the total system in which they are embedded. In fact, we can imagine the chain (3) extending indefinitely on both ends, with any successive triplet of mappings being an (M,R)-system, and in which any map could be either a metabolic component, a repair component, or a replication map, depending on which triplet was selected as primary. This relational result is explained in the algebraic-topological terms of ‘helical hierarchy’ in Section 6.21 of Louie (2009).

2. Morphisms

2.1 Mapping and its relational diagram

Let $f \in H(A, B)$ be a mapping from set A to set B (i.e. $f : A \rightarrow B$). When f is represented in the element-chasing version $f : a \mapsto b$ (where $a \in A$ and $b \in B$), its *relational diagram* may be drawn as a network with three *nodes* and two *directed edges*, i.e. a directed graph (or *digraph* for short):

$$f \xrightarrow{\quad} a \xrightarrow{\quad} b \quad (7)$$

The *hollow-headed arrow* denotes the *flow* from input (material cause) $a \in A$ to output (final cause) $b \in B$, whence the final cause of the mapping may be identified also as the hollow-headed arrow that terminates on the output:

$$\xrightarrow{\quad} b \quad (8)$$

The *solid-headed arrow* denotes the induction of or constraint upon the flow by the *processor* (efficient cause) f , whence the efficient cause of the mapping may be identified also as the solid-headed arrow that originates from the processor:

$$f \xrightarrow{\quad} \quad (9)$$

The formal cause of the mapping may be identified as the *ordered pair* $\langle \text{processor, flow} \rangle$ of the two kinds of arrows:



The processor and output relationship may be characterized ' f entails b ', denoted by:

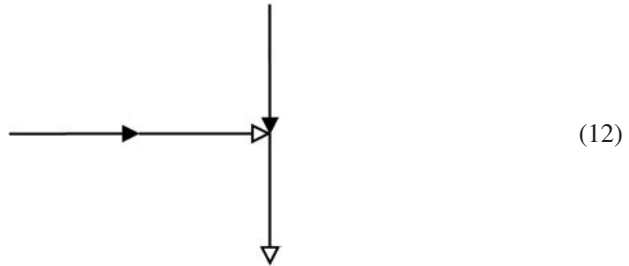
$$f \vdash b, \quad (11)$$

where \vdash is called the *entailment symbol*. The final cause b , the target of entailment \vdash and at the head of the hollow-headed arrow, is *that which is entailed*. A relational diagram is also called an *entailment network*.

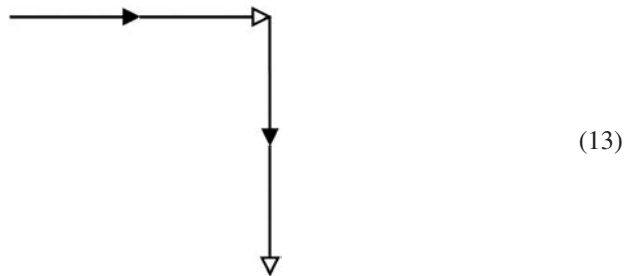
2.2 Composition

The relational diagrams of mappings may *interact*: two mappings, with the appropriate domains and codomains, may be connected at different common nodes. (For an in-depth study on the various modes of connections, see Chapter 5 of Louie 2009, and also Louie 2010.)

Composition of two mappings involves the combination of their digraphs in such a way that a final cause of one is relayed to become an ingredient of the other; i.e. when one mapping entails some component of the other. Two of the interactions of two mappings may be considered as compositions:

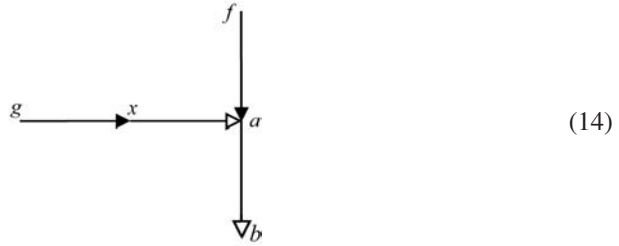


and:

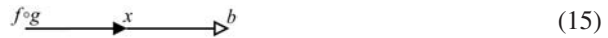


The relational interaction (12) arises when one has two mappings $f \in H(A, B)$ and $g \in H(X, A)$, whence *the codomain of g is the domain of f* . Let the element chases be $f : a \mapsto b$ (thus $f \vdash b$) and $g : x \mapsto a$ (thus $g \vdash a$), whence *the final cause of g is the material cause of f* . The relational diagrams of the two mappings connect at the common

node a as:



This *sequential composition* of relational diagrams represents the composite mapping $f \circ g \in H(X, B)$ with $f \circ g : x \mapsto b$, and has the abbreviated relational diagram:

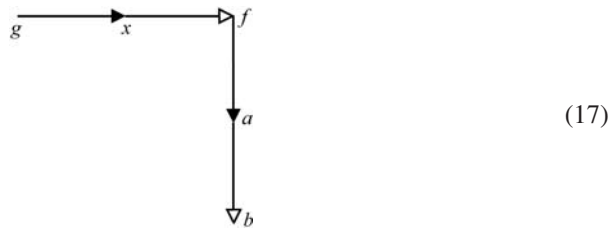


whence the corresponding entailment diagram is:

$$f \circ g \vdash b \tag{16}$$

($f \circ g$ entails b).

Next, the relational interaction (13) happens when one has two mappings $f \in H(A, B)$ and $g \in H(X, H(A, B))$, whence *the codomain of g contains f* . Because of this ‘containment’, the mapping g may be considered to occupy a higher ‘hierarchical level’ than the mapping f . Let the element chases be $f : a \mapsto b$ and $g : x \mapsto f$, whence *the final cause of g is the efficient cause of f* . Then one has the *hierarchical composition* of relational diagrams:



with the corresponding composition of entailment diagrams:

$$g \vdash f \vdash b. \tag{18}$$

Note that diagram (18) shows an iterative *entailment of an entailment*. A comparison of (16) and (18) reinforces the graphical differences of diagrams (12) and (13), and shows that sequential composition and hierarchical composition are different in kind: they are different both *formally* and in content.

3. Hierarchical cycle

For a collection of mappings in a formal system, their compositions may give rise to a very complicated pattern of inferential entailment in a network. The various network topologies are explored in Chapter 6 of Louie 2009.

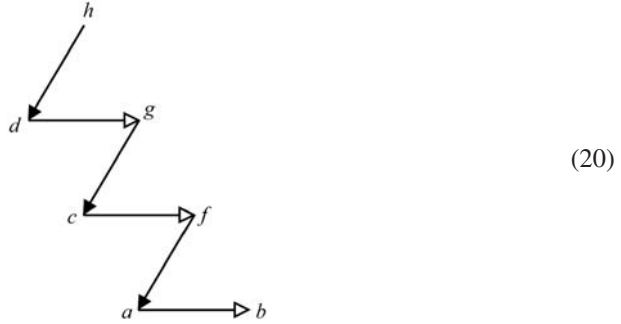
There is one mode of connection that is of special interest: when *two or more* hierarchical compositions are involved in a *cycle*, i.e. a closed path. (Note that a closed path in the directed graph sense means the arrows involved have a consistent direction.) This is called a *closed path of efficient causation*. In other words, a closed path of efficient

causation is an entailment cycle that contains two or more efficient causes. Both the hierarchy of containment and the cycle are essential attributes of this closure.

For example, consider three mappings from a hierarchy of hom-sets

$$f \in H(A, B), \quad g \in H(C, H(A, B)), \quad h \in H(D, H(C, H(A, B))). \quad (19)$$

Their hierarchical compositions form the relational diagram:



(where, naturally, $a \in A$, $b \in B$, $c \in C$, and $d \in D$). Now suppose that there is a correspondence between the sets B and $H(D, H(C, H(A, B)))$. Then an isomorphic identification between b and h may be made, and a cycle of hierarchical compositions results:



In diagram (21), one may say that the hierarchical compositions of the three maps $\{f, g, h\}$ are in *cyclic permutation*. The corresponding cyclic entailment pattern is:



One may say that the diagram (22) represents the *cyclic entailments*:

$$\{g \vdash f, h \vdash g, f \vdash h\}. \quad (23)$$

Formally, one has the definition as follows.

DEFINITION. A *hierarchical cycle* is the relational diagram in graph-theoretic form of a closed path of efficient causation.

4. Complex system and clef system

Note that in a hierarchical cycle (for example, arrow diagram (21)), there are *two or more solid-headed arrows* (since a closed path of efficient causation is defined as a cycle containing *two or more* hierarchical compositions). A hierarchical cycle is by definition

the formal-system representation (i.e. encoding) of a closed path of efficient causation in a natural system, so trivially one has the lemma as follows.

LEMMA. A natural system has a model containing a hierarchical cycle if and only if it has a closed path of efficient causation.

Because of this equivalence of a closed path of efficient causation in a natural system and a hierarchical cycle in its model, the term *hierarchical cycle*, although defined for formal systems, sometimes gets decoded back as an alternate description of the closed path of efficient causation itself. In other words, one may speak of a hierarchical cycle of inferential entailments as well as a hierarchical cycle of causal entailments.

DEFINITION. A natural system is *simple* if and only if it contains no closed path of efficient causation.

The class of natural systems *complementary* to simple systems is therefore given by the definition as follows.

DEFINITION. A natural system is *complex* if and only if it contains a closed path of efficient causation.

Equivalently, a natural system is *complex* if and only if it has a model that contains a hierarchical cycle.

In formal systems, closed paths of efficient causation, i.e. cycles of entailment, are manifested by impredicatives (or ‘self-references’). In science, where entailment means causality, causal cycles empower a rigorous study on the categories of final causation, whence on *function* and *anticipation*. To say that something is a final cause of a process is *to require the process to entail something*. Final cause thus requires something of its effect. Indeed, a final cause of a process must *entail the entailment of the process itself*. (See, in particular, the discussion on ‘immanent causation’ in the last few sections of Chapter 5 of Louie 2009.) It is this peculiar reflexive character of final causation that links it intimately to impredicative, hierarchical cycles, in which the hierarchical compositions are precisely ‘entailments of entailment’.

Note that a complex system only requires the *existence* of a hierarchical cycle that contains two or more processes. There may be many of its constituent processes that are not part of hierarchical cycles.

DEFINITION. A natural system is *closed to efficient causation* if its every efficient cause is entailed within the system.

Equivalently, a closed-to-efficient-cause system has a model in which *all* processes are involved in hierarchical cycles. The equivalence also allows the description *closed to efficient causation* to be used on formal systems.

Let me emphasize that the predicates contains ‘a closed path of efficient causation’ and ‘is closed to efficient causation’ are *not* equivalent. The class of systems that are closed to efficient causation forms a *proper subset* of the class of systems that contain a closed path of efficient causation. Members of the latter class, i.e. complex systems, are required to have only *some*, and not necessarily *all*, processes involved in hierarchical cycles.

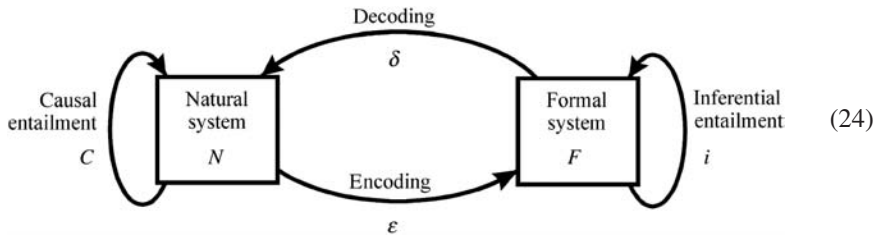
Instead of the verbose ‘closed-to-efficient-cause system’ or ‘systems that are closed to efficient causation’, in Louie and Poli (2011) we have introduced a new term ‘*clef* system’ (for *closed to efficient causation*) with the definition as follows.

DEFINITION. A natural system is *clef* if and only if it has a model that has all its processes contained in hierarchical cycles.

The word ‘clef’ means ‘key’; so this terminology has the added bonus of describing the importance of the class of *clef systems*. Analogously, a *clef* formal system is one that has all its mappings contained in hierarchical cycles. Thus, the class of clef systems forms a proper subset of the class of complex systems.

5. Functors

The most important notion in the definition of an anticipatory system is that of a *model*. A *modelling relation* is a commutative functorial encoding and decoding between two systems. The situation may be represented in the following canonical diagram:



in which the conditions:

$$i = \varepsilon(c) \quad \text{and} \quad c = \delta \circ i \circ \varepsilon \quad (25)$$

are satisfied. F is thence a *model* of N , and N is a *realization* of F .

A plethora of wondrous things follows from the establishment of a modelling relation between systems. I shall not get into them here; details may be found in Rosen (1985) and Chapter 4 of Louie (2009). The main conclusion, of course, is that one can use the inferential structure of the model to study that of its realization, to *impute* from theorems i in F an encoded hypothesis (via $\delta \circ i \circ \varepsilon$ in the diagram) back to causal entailments c in N . Note that while one may refer to the imputation $i \rightarrow c$ as ‘prediction’, the *predictive model* used in the definition of an anticipatory system requires more. (If ‘predictive model’ simply means this imputation, then it would have been a redundancy, since ‘all models predict’ in this sense.) A model is not *necessarily* ‘predictive’ in the temporal sense: time may not be explicitly involved in the sequencing of the congruence of inferential and causal entailments. A *predictive model* is one in which *time* (i.e. its ‘internal clock’) runs *faster than its realization*. (More on this in Section 7 below.) This extra qualification makes the special class of predictive models a proper subset of the collection of *all* models.

The basic problem of theoretical science is the establishment of relations between a real (physical, biological, social, ...) system and its models (or models that one can make of that system). These relations are also important in our understanding the behaviour of systems that contain models (of themselves, of their environments, of the future, ...) and use these models for purposes of control. The existence of internal models (anticipatory or otherwise) of self and environment is, indeed, an essential difference between biological systems and their complement set. It is in this sense that one may say that organisms are ‘model-based’. The study of organisms is thus the study of the relations between them and their models, whence relational biology. It is an unfortunate consequence of mechanism-based science that in reductionistic haste, the presence of models has often been neglected.

Now let us return to the canonical diagram (1) of an anticipatory system. I shall use the same symbols S , M , and E for the object, model, and effector systems, respectively, to denote their efficient causes. In other words, let each symbol represent the *processor* associated with the block (the ‘black box’) as well as the block itself. Then the entailment diagram for the anticipatory system is:



The maps labelled with circled numbers correspond to those in diagram (1). The map $\varepsilon : S \rightarrow M$, completing the cycle, is the *encoding* of the object system S into its model M . The entailment of the three maps $\{M, E, S\}$ in cyclic permutation renders this anticipatory system complex.

An anticipatory system has more structure in its entailment pattern than the cycle:

$$\{S \vdash M, E \vdash S, M \vdash E\}. \tag{27}$$

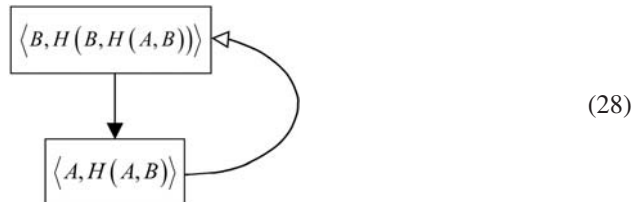
In particular, the model-updating map ③: $E \vdash M$, an inverse efficient cause, cannot be present in every hierarchical cycle. Thus, ‘an anticipatory system *must* be complex; a complex system *may* be anticipatory’.

6. (M,R)-network

I now have all the pieces with which to define an (M,R)-network. Let us revisit the simplest (M,R)-system (2), but with an alternate description.

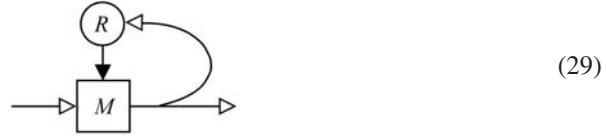
Metabolism may be considered an input–output system, with the mapping f representing the transfer function of the ‘block’, the domain A as the set of inputs, and the codomain B as the set of outputs. Thus, I may define a *metabolism component* as the formal system $M = \langle A, H(A, B) \rangle$. (For the purpose of this paper, I shall simply define a *formal system* as a pair $\langle S, F \rangle$, where S is a set and F is a collection of mappings with domain S . The reader may delve into this subject of categorical system theory in Chapter 7 of Louie 2009.)

Similarly, *repair* may be considered an input–output system, with the mapping Φ representing the transfer function of the block, the domain B as the set of inputs, and the codomain $H(A, B)$ as the set of outputs. Thus, I may define a *repair component* as the formal system $R = \langle B, H(B, H(A, B)) \rangle$. Then the model network diagram of this simplest (M,R)-system is:



With the addition of entailment arrows for environmental inputs and outputs, and the abbreviated representation by the symbols M and R of the components, I arrive at the simplest example of what I call an *(M,R)-network*, i.e. a network of metabolism and repair

components:



Network (29) is a modified form of the ‘abstract block diagrams’ that Rosen started using in his introductory (M,R)-systems papers (Rosen 1958, 1959), in which the emphasis was on the networks of metabolism and repair components.

I now compile these concepts in the definition as follows.

DEFINITION. *Metabolism and repair* are input–output systems that are connected as *components* into a network. They are formal systems with the following further category-theoretic structures.

- (a) A *metabolism component* is a formal system $M_i = \langle A_i, H(A_i, B_i) \rangle$.
- (b) A *repair component* is a formal system $R_i = \langle Y_i, H(Y_i, H(A_i, B_i)) \rangle$.
- (c) A *metabolism–repair network*, i.e. an *(M,R)-network*, is a finite collection of pairs of metabolism and repair components $\{(M_i, R_i) : i \in I\}$, connected in a model network with the following topology:
 - (i) the outputs of a repair component R_i are observables in $H(A_i, B_i)$ of its corresponding metabolism component M_i ;
 - (ii) the metabolism components may be connected among themselves by their inputs and outputs (i.e. by $B_k \subset A_j$ for some $j, k \in I$);
 - (iii) repair components must receive at least one input from the outputs of the metabolism components of the network (i.e. $Y_i = \prod_{k=1}^n B_{i_k}$ with $n \geq 1$ and where each $i_k \in I$).

Note that the connections specified in (i)–(iii) are the *requisite* ones; an (M,R)-network may have additional interconnections among its components and with its environment.

Finally, I can give the formal definition as follows.

DEFINITION. A *metabolism–repair system*, i.e. an *(M,R)-system*, is an (M,R)-network that is closed to efficient causation.

Stated otherwise, an (M,R)-system is an (M,R)-network that is also a clef system.

Note that the Axiom of Anticipation (that life is anticipatory) in Section 1.1 and the result established in Section 5 (that an anticipatory system is complex) lead syllogistically to

Rosen’s Theorem: An organism must be complex; a complex system may (or may not) be an organism.

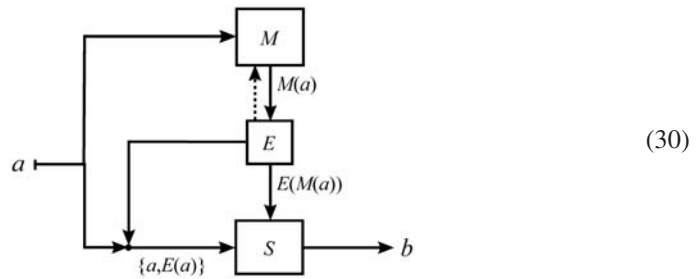
(cf. Chapter 11 of Louie 2009). Likewise, the Postulate of Life [that life is a realized (M,R)-system] from Section 1.2, together with the formal definitions of complex system and (M,R)-system, consistently lead to the same Rosen’s Theorem.

7. Prediction

It should be clarified that ‘anticipation’ in Rosen’s usage, embodied in the ‘predictive model’, does not refer to an ability to ‘see’ or otherwise sense the immediate or the distant

future – there is no prescience or psychic phenomena suggested here. Instead, Rosen suggests that there must be information about self, about species, and about the evolutionary environment, encoded into the organization of all living systems. He observes that this *information*, as it behaves through time, is capable of acting causally on the organism’s present behaviour, based on the relations projected to be applicable in the future. Thus, while not violating time established by external events, organisms seem capable of constructing an internal surrogate for time as part of a model that can indeed be manipulated to produce anticipation. In particular, this ‘internal surrogate of time’ must run *faster than real time*. It is in this sense that degrees of freedom in internal models allow time its multi-scaling and reversibility to produce new information. The predictive model in an anticipatory system must not be equivocated to any kind of ‘certainty’ (even probabilistically) about the future. It is, rather, an assertion based on a model that runs in a faster timescale. The future still has not yet happened: the organism has a *model* of the future, but not definitive knowledge of future itself. Indeed, the predictive model may sometimes be wrong, the future may unfold very differently from the model’s predictions, and the consequences of the mismatch may be detrimental to the anticipator.

Let me now reformulate the canonical diagram (1) of an anticipatory system. If one traces the path of an input element a , the diagram becomes:



and the corresponding output b will satisfy the functional equation:

$$b(t) = S[a(t), E(a(t)), E(M(a(t)))]. \tag{31}$$

Within such generality, it is easy to see that it is possible to define many different timescales. In particular, the internal surrogate of time in $M(a(t))$ must run faster than the external time t for M to be a *predictive model*.

8. Internal timescales in an (M,R)-system

In the simplest (M,R)-network (29), since there is only one metabolism component and one repair component, the domain of the repair component is restricted to the codomain of the metabolism component, i.e. $Y = B$ in requisite connection (iii) of the definition of (M,R)-network. But with more components, for each repair component R_i there are more choices for its domain $Y_i = \prod_{k=1}^n B_{ik}$. In particular, it is entirely feasible that in the product of codomains, none of the B_{ik} is B_i itself. Stated otherwise, while R_i repairs M_i , the replenishment of R_i need not depend on M_i .

This brings me to the topic of dependency of metabolism and repair components, with the definition as follows.

DEFINITION. A metabolism component in an (M,R)-network is *reestablishable* if the network has the capacity to replace it in the event of its absence. Otherwise, the metabolism component is *non-reestablishable*.

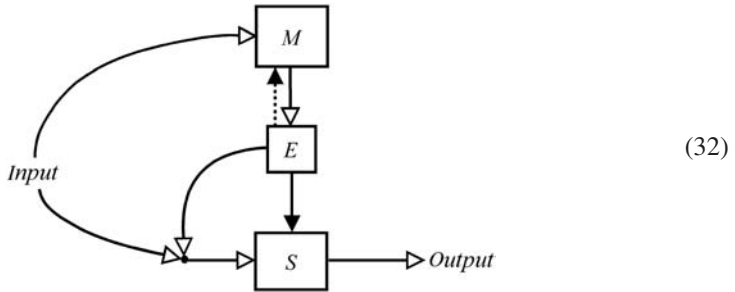
In graph-theoretic terms, a metabolism component is reestablishable if and only if there is no directed path from it to its corresponding repair component. Thus, in particular, the metabolism component M in the simplest (M,R)-network (29) is non-reestablishable. Rosen (1958, 1959, 1972) and Chapter 13 of Louie (2009) all contain explorations into this fascinating topic of (M,R)-systems. So I shall not repeat them here. I shall, instead, specialize on the temporal aspects.

If the concept of separate internal timescales for metabolism and repair components is introduced, then reestablishability takes on a new characterization. Even when a directed path does lead from a metabolism component M_i to its corresponding repair component R_i , the component M_i may still be replenished by its repair component R_i if R_i has a longer operational lifetime. In such a case, M_i may be repaired by R_i before R_i perishes due to lack of input from M_i , but then once M_i is repaired the input line to R_i is reestablished, so that no irreversible damage occurs. In other words, the presence of ‘finite time lags’ (often manifest in natural systems as *hysteresis*) allows M_i to be rebuilt into the (M,R)-network although M_i is graph-theoretically non-reestablishable.

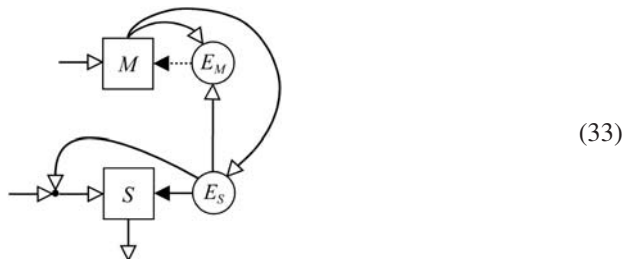
We have, of course, just encountered the multiple scaling of time in the previous section, in the context of anticipatory systems. When metabolism and repairs components have different timescales, the resulting (M,R)-network therefore behaves temporally like an anticipatory system. I shall next show the *converse* that the entailment pattern of an anticipatory system may be represented as a relational network of metabolism and repair components, thus completing an alligation of the two categories of objects with which we began this paper in Section 1.

9. Effectors as repair components

The simple anticipatory system (*cf.* diagrams (1) and (30)) may be redrawn in graph-theoretical form as the model network:



Note that the set E of effectors functionally entails both the system S and the internal predictive model M . If I fractionate E into the functional components E_S that acts on S and E_M that acts on M , the model network (32) becomes:



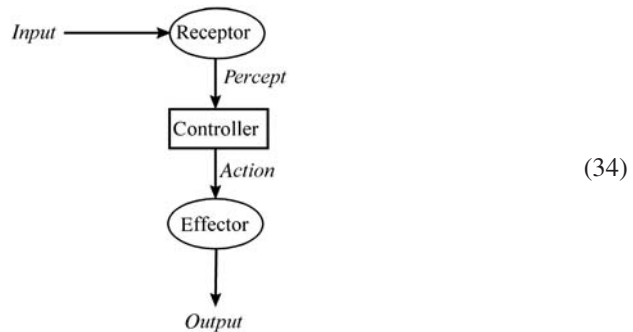
which is recognizably an (M,R)-network $\{(S, E_S), (M, E_M)\}$, with two pairs of metabolism and repair components. The pair (M, E_M) is the internal predictive model of the anticipatory system (33), and has an internal timescale that runs faster than real time (or at least faster than the clock of (S, E_S)).

10. The (M,R)-network of an anticipatory system

It is an opportune time to further explicate the concept of *effector*, since it assumes the crucial role of repair component. With the iconic diagram (1), effectors were introduced tersely in Rosen (1974), thus

We shall for the present suppose simply that the system M is equipped with a set E of *effectors*, which allow it to operate either on S itself, or on the environmental inputs to S , in such a way as to change the dynamical properties of S .

While *effector* may not be a common-usage term, it is in fact standard in *control theory*, in which *receptor* and *controller* are the other two key components. The relationship among the triumvirate is represented in the following control block diagram:



A *receptor* (which may also be called *sensor* in context) interacts with the environment (i.e. accepts inputs) and encodes into percepts, information that is passed on to the coordinating device. A *controller* (also *coordinator*, *transducer*) maps the distinctions made by the receptor into actions. An *effector* implements a semantic relation and carries out the instructed actions, thus produces an effect (i.e. releases outputs) into the environment.

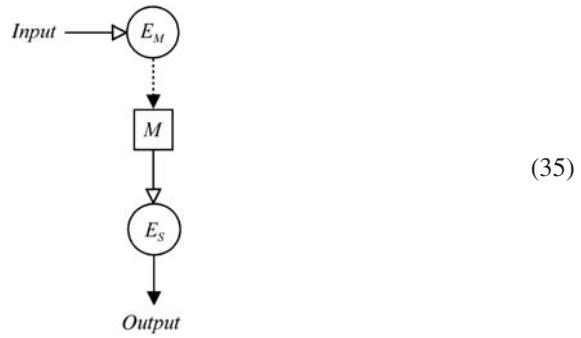
For a simple example, consider the withdrawal reflex of the human nervous system. The input may be the stimulus of touch on a sharp object. The receptor is the pain sensory neuron. The controller is the central nervous system. The effector is the muscle. The output is the response of a movement of withdrawal.

The homeostatic regulation of body temperature through negative feedback offers another example. Here, the input may be the stress of hyperthermia (i.e. overheating). The receptor is the heat sensory neuron. The controller in this case is the hypothalamus. The effectors are physiological responses such as increased activity in sweat glands and increased blood flow to the skin. The effect is perspiration, which evaporates whence cooling the skin.

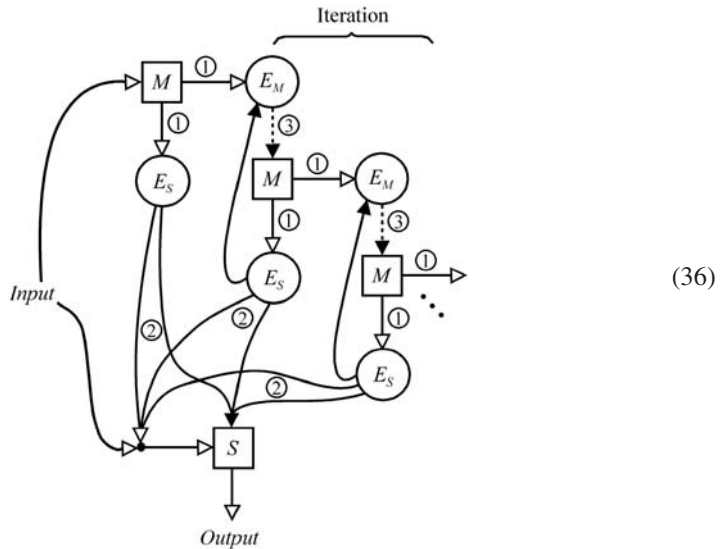
From its definition and from the examples, one sees that an *effector* carries out the actual response process, whence occupies the central role of ‘generating the efficient causes’; i.e. *repair* in the (M,R)-network sense. If one revisits the definitions of the ‘anticipatory modes of behaviour of organisms’ and ‘anticipatory system’ in Section

1.1, one sees that the ‘internal predictive model’ is the *controller*. It is, however, crucial to remember that what defines an anticipatory system is *not* just the *existence* of the predictive model – there are *two* indispensable ingredients: (a) predictive model and (b) response to the prediction. It is interesting to note that these two components have their analogies in an (M,R)-network precisely as (a) metabolism and (b) repair.

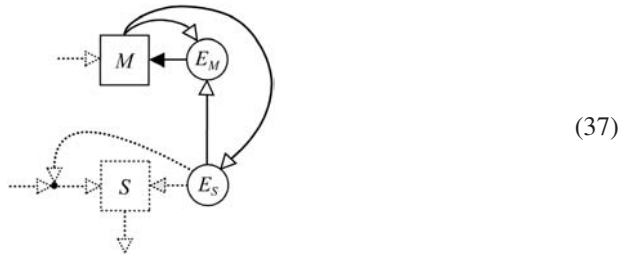
A control block diagram with the predictive model M as the controller would look like this:



The *response* of the anticipatory system in accord with the model’s prediction means that the control system unit (35) is *iterated* (for as many times as deemed necessary). A control block diagram of an anticipatory system as an (M,R)-network (33) may therefore be drawn thus (the circled numbers correspond to those in diagram (1)):



The iteration in relational diagram (36) corresponds to an expansion of the entailment cycles of diagram (33), highlighted below in solid lines:

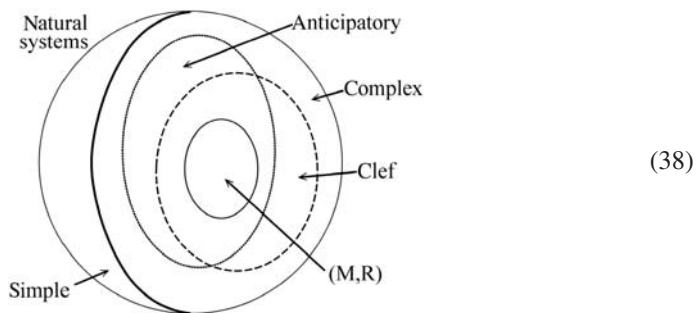


Relational diagrams (35) and (37), then, represent, respectively, the two defining features of an anticipatory system: (a) predictive model and (b) response to the prediction.

11. The relational taxonomy

Rosen created the theory of anticipatory systems as a stepping-stone towards the ultimate goal of the characterization of life. There is no question that the subject of *anticipation* itself is of independent interest, far-reaching, and tremendously worthy of study. It must, however, be remembered that the *raison d'être* of biology, hence of our relational approach to the subject, is *life* itself. Stated otherwise, anticipatory behaviour is a necessary, but not sufficient, condition for life. Live is anticipatory, but not all anticipatory systems are living. Anticipation (or even the slightly more general ‘model-based behaviour’) can only be used as an explanatory principle to some, but not all, characteristics of living systems. Indeed, the behaviour of a living system is more often characterized by its *impredicativity* (i.e. *complexity*), a property due to the existence of a closed path of efficient causation, a property that an anticipatory system happens to share. An anticipatory system is impredicative; an impredicative system may (or may not) be anticipatory. Attributing too much to anticipation instead of impredicativity would become an equivocation.

It is fitting to close this paper with a Venn diagram of the various classes of natural systems that I have explicated herein:



Note

1. Historical note: On 18 October 1972, the then-newly founded *International journal of general systems* received *four* papers submitted by Robert Rosen (all prepared when he was a Visiting Fellow at the Center for the Study of Democratic Institutions in Santa Barbara, CA, during the academic year 1971–1972). Their final forms were accepted on various dates in 1973–1974 for publication, and all appeared in Volume 1 (1974) of this journal (indeed, one in each of the four issues). The idea of anticipatory systems was discussed in two of these papers. Although the other paper (‘Some temporal aspects of political change’), since it was accepted sooner, actually

appeared in print before the ‘Four fuzzy concepts’ paper, the latter was written earlier, whence had precedence.

Notes on contributor



A.H. Louie is a mathematical biologist, in the relational biology school of Nicolas Rashevsky and Robert Rosen. Louie’s book on the subject, *More than life itself: a synthetic continuation in relational biology* was published in 2009 by ontos verlag. (George Klir has recently reviewed the book in this journal, Vol. 39, No. 7, October 2010, pp. 793–796.) Louie’s premier interest is in pure mathematical biology: conception and abstract formulations. His two major research topics are category-theoretic aspects of living systems and a phenomenological calculus based on multilinear algebra. Louie received his B.Sc. (Hons) degree with a double major in Biology and Mathematics and his MA degree in Pure Mathematics, in 1978 and 1979, respectively, from the University of Western Ontario (London, Canada). Then, in 1981, he received his PhD degree in Mathematical Biology, under the supervision of Robert Rosen, from Dalhousie University (Halifax, Canada). Rosen wrote the monograph *Anticipatory systems* in the first 6 months of 1979; Louie became Rosen’s PhD student shortly after the first draft had been completed, and was one of the first to read it. Louie still possesses a copy of the original annotated manuscript, typed by Rosen himself on his then-state-of-the-art IBM Selectric II typewriter.

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